Order estimates for uniform approximations by Fourier sums of the classes of convolutions of periodic functions of not high smoothness

A.S. Serdyuk[1], **T.A. Stepaniuk**[2]

[1] *Institute of Mathematics of NASU, Kyiv, Ukraine, serdyuk@imath.kiev.ua;*
[2] *Lesya Ukrainka Eastern European National University, Lutsk, Ukraine, tania_stepaniuk@ukr.net*

By $\mathfrak{M}$ we denote the set of continuous, convex downward, positive functions $\psi(t)$, $t \geq 1$, that vanish at infinity. For every function $\psi \in \mathfrak{M}$ (see e.g., [1]) we introduce the characteristic $\alpha(\psi;t) := \frac{\psi(t)}{t|\psi'(t)|}$, $\psi'(t) := \psi'(t+0)$ and we denote $\mathfrak{M}_0 = \{\psi \in \mathfrak{M} : \exists K > 0 \ \forall t \geq 1 \ \ \alpha(\psi;t) \geq K\}$.

Let $C^{\psi}_{\beta,p}$ be the class of 2π–periodic functions f, represented by the convolutions $f(x) = \frac{a_0}{2} + \frac{1}{\pi} \int\limits_{-\pi}^{\pi} \Psi_\beta(x-t)\varphi(t)dt, \ \varphi \perp 1, \ \|\varphi\|_p \leq 1, \ 1 \leq p < \infty, \ \beta \in \mathbb{R}, \ a_0 \in \mathbb{R},$ where $\Psi_\beta(t) = \sum\limits_{k=1}^{\infty} \psi(k) \cos\left(kt - \frac{\beta\pi}{2}\right)$, $\Psi_\beta \in L_{p'}$, $\frac{1}{p} + \frac{1}{p'} = 1$, $\psi(k) > 0$.

We consider the problem of finding the exact–order estimates of quantities $\mathcal{E}_n(C^{\psi}_{\beta,p})_C = \sup\limits_{f \in C^{\psi}_{\beta,p}} \|f(\cdot) - S_{n-1}(f;\cdot)\|_C$, where $S_{n-1}(f;\cdot)$ are Fourier sums of order $n-1$ of the function f.

Theorem 1. *Let $g_p(t) := \psi(t)t^{\frac{1}{p}} \in \mathfrak{M}_0$ and $\sum\limits_{k=1}^{\infty} \psi^{p'}(k)k^{p'-2} < \infty$, $1 < p < \infty$, $\frac{1}{p} + \frac{1}{p'} = 1$. Then for arbitrary $n \in \mathbb{N}$ and $\beta \in \mathbb{R}$ the correlations are true:*

$$K^{(1)}_{\psi,p}\Big(\sum_{k=n}^{\infty} \psi^{p'}(k)k^{p'-2}\Big)^{\frac{1}{p'}} \leq \mathcal{E}_n(C^{\psi}_{\beta,p})_C \leq K^{(2)}_{\psi,p}\Big(\sum_{k=n}^{\infty} \psi^{p'}(k)k^{p'-2}\Big)^{\frac{1}{p'}},$$

where $K^{(1)}_{\psi,p} = \frac{1}{\xi(p)}\left(\frac{\underline{\alpha}_1(g_p)}{p' + \underline{\alpha}_1(g_p)}\right)^{\frac{1}{p}}, \quad K^{(2)}_{\psi,p} = \frac{1}{\pi}\xi(p')\Big(1 + \frac{p'}{\underline{\alpha}_1(g_p)}\Big)^{\frac{1}{p'}}$, $\underline{\alpha}_1(\psi) := \inf\limits_{t \geq 1} \alpha(\psi;t)$, *and* $\xi(p) := \max\Big\{4\Big(\frac{\pi}{p-1}\Big)^{\frac{1}{p}}, \ \ 14(8\pi)^{\frac{1}{p}}p\Big\}$.

[1] A. I. Stepanets, *Methods of Approximation Theory*, VSP: Leiden, Boston, 2005.