

Some Properties of Orthogonal Wavelets Based on Jacobi Polynomials

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In this study we investigate wavelets based on Jacobi polynomials. We use the definition of n -th order wavelet as in [1]:

$$\psi_{n,r}(x) = \sum_{k=n+1}^{2n} P_k(t_r)P_k(x) \quad (1)$$

for some fixed set of parameters t_1, t_2, \dots, t_n . The definition of n -th order Jacobi polynomial depending on the parameters α, β is as follows

$$P_n^{(\alpha,\beta)}(x) = \frac{\Gamma(\alpha + n + 1)}{n!\Gamma(\alpha + \beta + n + 1)} \sum_{m=0}^n \binom{n}{m} \frac{\Gamma(\alpha + \beta + n + m + 1)}{\Gamma(\alpha + m + 1)} \left(\frac{x-1}{2}\right)^m,$$

where $\Gamma()$ is a gamma function.

In this investigation we consider the following questions regarding wavelets (1):

1. **Linear dependence.** For which $t_r, r = 1, \dots, n$, the wavelets $\psi_{n,r}(x)$ are linearly dependent?
2. **Orthogonality.** For which $t_r, r = 1, \dots, n$, $\psi_{n,r}(x)$ are orthogonal?
3. **Riesz stability.** For which $t_r, r = 1, \dots, n$, the minimum Riesz stability constant is achieved?

This investigation is a joint work with Professor Juergen Prestin (Universität zu Lübeck, Institut für Mathematik, Lübeck, Germany).

- [1] B. Fischer, J. Prestin, Wavelets based on orthogonal polynomials, *Mathematics of computation*, **66** (1997), p. 1593–1618.