The best orthogonal trigonometric approximations of functions of the classes $L^{\psi}_{\beta,1}$

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The paper is devoted to the study of the approximation of periodic functions of one variable of the classes $L^{\psi}_{\beta,1}$ in the space $L_q,\ 1<q<\infty$.

Let B be the set of functions ψ satisfying the following conditions: 1) ψ are positive and nonincreasing; 2) exists a constant $C>0$ such that $\frac{\psi(t)}{\psi(2t)} \leq C,\ t\in\mathbb{N}$.

Let L_q be the space of 2π-periodic functions f with the usual norm. We denote by

$$e^{\perp}_m(L^{\psi}_{\beta,1})_q = \sup_{f\in L^{\psi}_{\beta,1}} \inf_{\Theta_m} \Big\| f(\cdot) - S_{\Theta_m}(f,\cdot)\Big\|_q$$

the best orthogonal trigonometric approximations of the classes $L^{\psi}_{\beta,1}$, where

$S_{\Theta_m}(f,x) = = \sum\limits_{k=1}^{m} \hat{f}(n_k)e^{in_k x},\ \ \Theta_m\subset\mathbb{N}$ — a finite set containing m elements and $\hat{f}(k) = = \frac{1}{2\pi}\int\limits_{-\pi}^{\pi} f(t)e^{-ikt}dt$ — Fourier coefficients of f.

We prove the following

Theorem 1. *Let* $1<q<\infty,\ \psi\in B,\ \beta\in\mathbb{R},\ \Delta^2\left(\frac{1}{\psi(t-1)}\right)\geq 0,\ t=\overline{1,m-1}$, *or* $\Delta^2\left(\frac{1}{\psi(t-1)}\right)\leq 0,\ t=\overline{1,m-1}$, *where* $\Delta^2\left(\frac{1}{\psi(t)}\right) = \frac{1}{\psi(t)} - \frac{2}{\psi(t+1)} + \frac{1}{\psi(t+2)}$ *and exists* $\varepsilon>0$*:* $\psi(t)t^{1-\frac{1}{q}+\varepsilon}, t\in\mathbb{N}$ *are nonincreasing. Then the following estimate is true:*

$$e^{\perp}_m(L^{\psi}_{\beta,1})_q \asymp \psi(m)m^{1-\frac{1}{q}}.$$